Design and feedback for a sequence of lessons in geometry and numeracy using the Causal Connectivity Framework

Where Mathematics meets play in the Lower Primary

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Introduction

The new Ofsted framework has emphasised the quality of the curriculum (Ofsted, 2019). Although Ofsted does not stipulate a preference for any of the three possible curriculum intents (knowledge-led approach, knowledge-engaged approach and skills-led curricula), one aspect of quality assurance for curricula is related to 'retrieval of core knowledge baked into the curriculum' (Spielman, 2018). How well the curriculum is implemented depends heavily on there being well-taught and appropriately designed sequenced content, especially targeting the core knowledge. In this article, we will demonstrate how to use the Causal Connectivity Framework (see Fig.1) guided a sequence of lesson design from knowledge-engaged approach. We choose one of the core knowledge at lower primary setting, the geometry topic of 2D and 3D shapes as the content, while the main foci is to improve young pupils' mathematics reasoning skills. We will also exemplify how curriculum requirements in this case can be enriched though play which fits with lower primary students' cognitive characteristics.

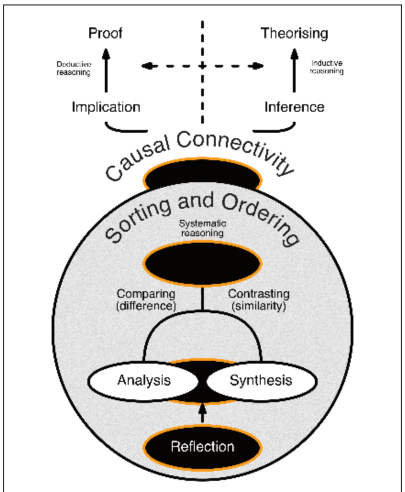

Figure 1 Causal Connectivity Framework

The UK has increasingly promoted culturally cultivated play, especially in lower primary level (for example, the Scotland's Play Strategy). Play is seen as an important part of encouraging and enjoying the learning journey. Although there are, of course, different types of play – such as games, role play, and free-play - this article will take the free-play approach at the beginning in setting up play-based activities as an essential stimulus, and later asking pupils explain the logic behind their actions for the free-play in order to develop pupils' reasoning ability. These play-based activities fit with Causal Connectivity Framework form part of Phase 2 (Analysis and Synthesis), and Phase 3 (Sorting and Ordering), laying the foundation for pupils to develop causal links for new knowledge. We will also outline feedback from pupils, classroom teachers and school senior leaders six months after the implementation. The purpose of this paper is to draw more attention to young pupils' mathematics reasoning, and to contribute to the discussion on how to embed maths-rich play-based activities into curriculum requirements.

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This sequence of the lessons, which we call The Snowflake Bentley Project, has been introduced to Year 2 and Year 3 pupils as a way of improving 'thinking habits' in reasoning. The thinking habit it aims to instil is divided into two steps: first analysis and synthesis, and then sorting and ordering through a series of play-based activities. Furthermore, the explicit teaching of reasoning to all pupils from lower primary setting onwards could be beneficial. In this way, pupils with all levels of academic achievement are given opportunities and support to develop through these well-designed play-based activities. The carefully and strategically chosen activities build upon effective mathematical talk in the classroom so that pupils are able to work collaboratively and record their thinking (Meli & North, 2018). As a result, pupils can explore the relevant issues for themselves and develop their ability to conjecture and explore their own logical thinking approaches. This resonates with the principle of reasoning mathematically, one of the three National Curriculum aims.

The arrangement of these activities into different lessons emerges from the clear learning objectives, and the activities fulfil these two aspects: (1) inquiry activities, characterised as open, investigative, and analytical; and (2) engagement activities, which are enjoyable, and attention-grabbing (Foster & Inglis, 2018). The activities are placed in a particular order so that the aspects that the inquiry brings out become increasingly challenging. The play-based activities allow pupils to explore, test and apply knowledge, following their own lines of enquiry. Here, we present how the play-based activities act as an integrative learning experience for the specific mathematical objectives at their heart.

Lesson sequence

We present how the two areas in geometry are interwoven into two areas in numbers i.e. properties of shapes and position and direction are connected to number and place value and addition and subtraction. These are all elements of core knowledge in lower primary settings.

Sorting shapes into different categories is commonly used as an initial task. Tracey (2016), for example, describes how sorting different shapes leads to the next task of constructing triangles as a means of approaching the topic of tessellations. In this case, we use the same starting and ending points to demonstrate connections of knowledge within maths. Furthermore, in the third lesson, a literacy task is also added to enrich the learning.

The main resource used here is a set of shapes: equilateral triangle, square, regular pentagon, and regular hexagon (see Fig 2). All are commonly available in primary schools, although in this case the shapes have been specifically designed to ensure they fit together as shown in Fig 2. This resource is seen as a model of deductive reasoning rather than only as something interesting and enjoyable for pupils to handle. It can also be seen as an example of concrete representation that eventually moves towards pictorial and abstract representation.

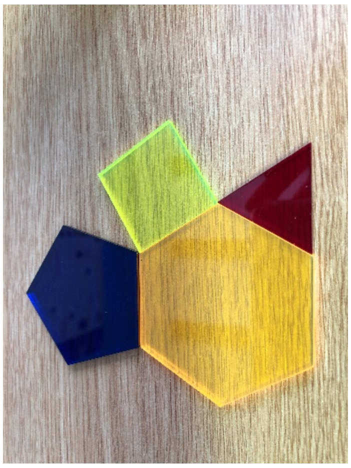
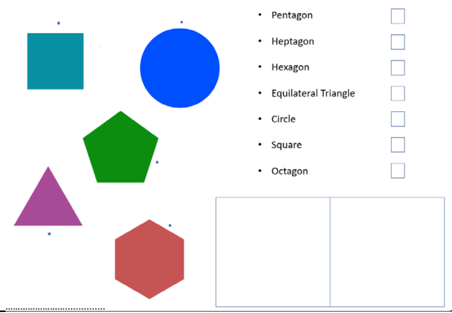
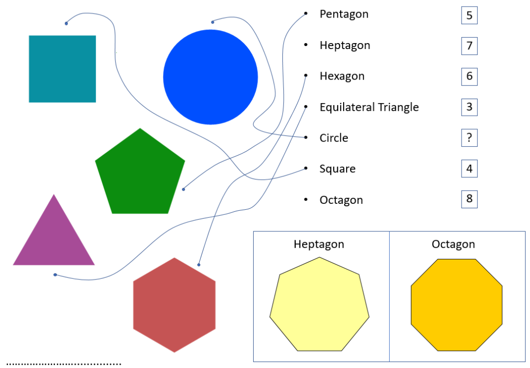
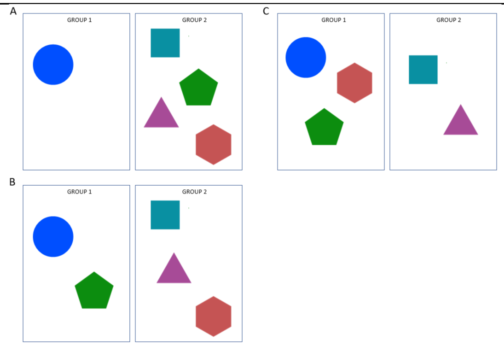
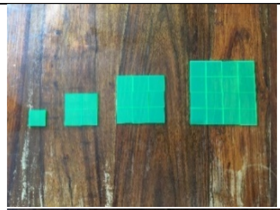
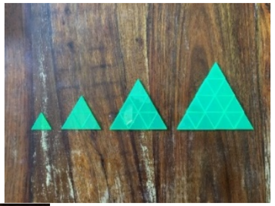
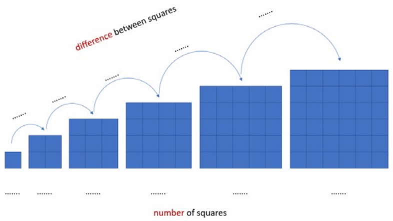
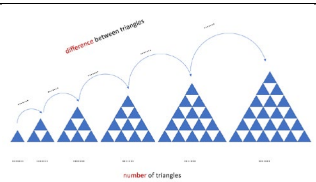


Figure 2 A set of shapes as concrete representation

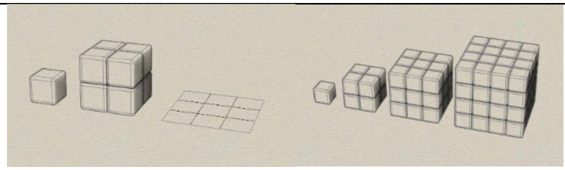
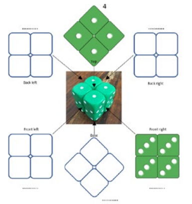
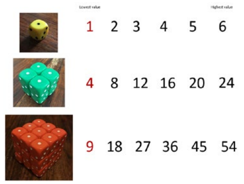
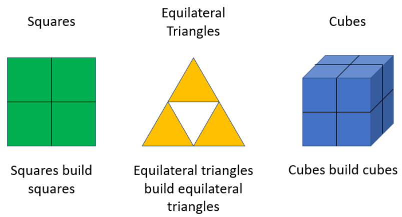
Lesson 1

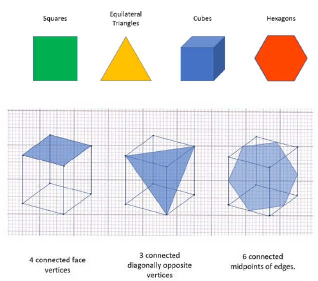
Before the lesson starts, several sets of shapes are placed on the pupils' tables. Pupils are likely to start playing with them, and most unconsciously try to create tessellation patterns. We use the following tables to illustrate the progression planned for in the learning sequence:

Play-based activities		Aims of the activities:	Causal Connectivity framework	National Curriculum requirements for Year 2 and Year 3
Can you link the shape with their proper names?		To reflect on the common shapes pupils have learned	Phase 1: Relevance	
Can you draw the other two shapes?		To raise awareness of the difference between regular and irregular shapes through drawing the heptagon and the octagon		Year 2 Geometry, properties of shapes: 'identify and describe the properties of 2D shapes, including the number of sides'. (Department for Education, 2013, p. 15)
Do you think these shapes are in the correct order?		To encourage the use of maths language	Phase 2: Analysis and Synthesis	Year 2 Geometry, properties of shapes: 'compare and sort' common 2D shapes based on their properties and use vocabulary precisely, such as sides (Department for Education, 2013, p. 15).

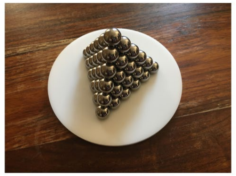
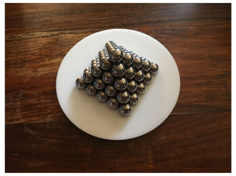
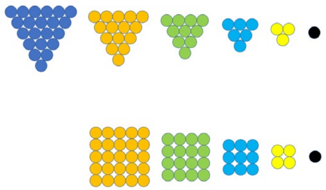
Here are three different groupings; can you explain why they have been grouped this way?		To introduce the concept of tessellation, and self-similarity	Phase 3: sorting and ordering	
How can we use squares to build larger squares?	  	To use the concept of tessellation; to focus on changes of sides, and changes of areas. Concept check notion of self-similarity.		Year 2 Geometry, position and direction: ‘order and arrange combinations of mathematical objects in patterns and sequences’. (Department for Education, 2013, p. 16)
How many squares are there in each square and how do they grow?		To connect shapes with number sequences, especially square numbers and triangular numbers	Phase 4: Causal Connectivity	
How many triangles are there in each triangle and how do they grow?				

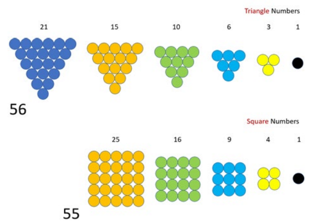
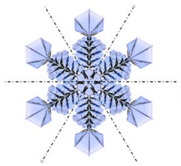
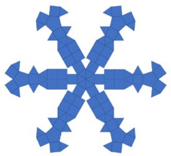
Lesson 2

Play-based activities		Aims of the activities	Causal Connectivity framework	National Curriculum requirements for Year 2 and Year 3
How many cubes are required to build a cube in the 3rd footprint? 4th? Count or calculate?		To transition from 2D to 3D shape and utilise previous concepts	Phase 1: Relevance	
Could you please build this model? Can you fill in the missing faces on the diagram and the total number?	 	To increase dexterity and observation and improve 3D visualisation.	Phase 2: Analysis and Synthesis	Year 3 Number, number and place value: 'count from 1 in multiples of 4'. (Department for Education, 2013, p. 18)
What would happen to the third cube?		To link physical and visual with abstract (basic number sequences)	Phase 3: Sorting and ordering	Year 3 Number, number and place value: 'solve number problems and practical problems involving these ideas'. (Department for Education, 2013, p. 18)
What is the connection?	 Square Numbers: Triangle Numbers: Cube Numbers:	To link with the concept of self-similarity in 3D To introduce the concept of lattice (which is a requirement for understanding crystals & crystal growth.)	Phase 4: Causal Connectivity	

Are there any different connections?		To consolide, via relationships, many into one. I.e. Relate all of the main shapes and objects to a cubic lattice.		
Let's make a cube together; can you find the triangle and the hexagon?	  	Constructing cubes and cubic lattices		Year 3 Geometry, properties of shapes: 'make 3D shapes using modelling materials, recognise 3D shapes in different orientations and describe them'. (Department for Education, 2013, p. 22)

Lesson 3

Play-based activities		Aims of the activities	Causal Connectivity framework	National Curriculum requirements for Year 2 and Year 3
Which has more? Count or calculate?	 	Understand how geometry plays a role in self-assembly (crystal growth)	Phase 2: Analysis and Synthesis	Year 2 Number, Number and place value: 'identify, represent and estimate numbers using different representations' (Department for Education, 2013, p. 11)
		Link previous activity to number work and number patterns	Phase 3: sorting and ordering	

		To link to triangle and square number sequences, using the strategy of addition	Phase 4: Causal Connectivity	Year 2 Number, addition and subtraction: ‘solve problems with addition and subtraction, applying their increasing knowledge of mental and written methods’. (Department for Education, 2013, p. 12)
Literacy of Snowflake: A story book about Bentley’s contribution towards science and photography	 	Literacy and numeracy (cross-curricula). What is a crystal? Drawing together of previously introduced concepts.		
How can we use sets of regular shapes – triangles, squares, and pentagons – to make a snowflake?	 	Back to play…but now making more complex patterns and arrangements based on symmetries.		

Evaluating the lessons

A thorough evaluation process was conducted to develop an understanding of how this play-based learning curriculum worked on its initial implementation and to suggest improvements for how it could be used in the future. This aimed to capture the impact on the pupils involved, their teachers and senior leaders. Three types of semi-structured, one-to-one, cross-sectional case study interviews were conducted in all three participating schools. Pupils' interviews, followed up by a 10-minute questionnaire, aimed to capture changes in their knowledge and understanding, and their confidence or interest in mathematics. Teacher interviews provided an opportunity to observe changes in professional pedagogy and practice. Interviews with senior leaders explored perceived impacts on wider thinking on curriculum planning. The interviews were conducted by the first and third authors. The ethics approval was obtained from School of Education Durham University on 4th May 2018, prior to this evaluation stage.

Student feedback on the core knowledge:

The majority of the pupils (79 out of 117, 68 percent) reported that the lessons were related to 'shapes' and the 'book' (i.e. Snowflake Bentley by Jacqueline Briggs Martin which was introduced in the third lesson). Nearly half of them (51 out of 117, 44 percent) were able to recall the concrete representations or manipulatives used in the three lessons, such as dice, ball bearings, rocks and crystals. A few recalled the formal mathematics terms from the project (ignoring some spelling errors): 18 out of 117 pupils (15 percent) mentioned the words 'triangle' and 'square', while 10 out of 117 (8.5 percent) mentioned 'hexagon'.

The notions of tessellation or symmetry were reported as the key ideas by 17 pupils (14.5 percent). These words were generally not spelled correctly; 'semetrical' and 'tesselate' were common errors. When asked about work on numbers from the lessons, various answers were given: 10 percent of the Year 2/3 pupils reported that the focus was on making patterns or big shapes.

Even though those pupils were not able to recall the specialist vocabulary introduced (i.e. array, self-similarity, tessellation, and lattice) without prompting, many recalled the ideas behind them. Once re-introduced to this specialist vocabulary, they were frequently able to use it appropriately in context. For example, having revisited the meaning of self-similar and tessellation in a 2D context, when asked: "Cubes build bigger cubes so cubes are..?" one child, for example, was able to correctly complete the sentence with "self-similar". Likewise, having revisited the meaning of tessellation, the same child correctly identified that neither circles nor pentagons tessellate and why: "they leave gaps".

Pupil feedback on feelings about the lessons:

70 out of 117 pupils (60 percent) reported that their feeling about the lessons was 'happy', and 45 of them (38.5 percent) felt 'excited'. 40 of them (34 percent) expressed mixed feelings, such as 'happy and confused', 'happy and scared', 'happy, excited, worried', or 'a little bit sad and angry and happy at the end'. None of them reported only negative feelings. 83 out of 117 pupils (71 percent) would recommend the lessons to other pupils as they found them 'interesting', 'amazing' and 'fun'. 12 of them (10 percent) reported that they would not recommend them as they were boring, or not fun.

Teacher feedback

From a pedagogical perspective, the teachers felt that the pilot activities led them to engage in lots of open-ended questioning, more so than is the norm in everyday mathematics lessons. It also provided an ideal opportunity to blend literacy with mathematics through the use of specialist vocabulary. One teacher commented that in terms of professional development, the project may have had more impact on teachers' pedagogy if they had engaged with the gap tasks in class, or been present in the original lessons.

Senior Leader feedback

One head teacher commented that this project had enabled the school to understand their learners better, reassessing their potential by making their learning visible in a way not seen within a traditional classroom setting. The project was felt to be strongly aligned with the 'mastery' approach in that it let children work collaboratively, exploring and building their own ideas:

> "Learning how to be inquisitive; learning it's alright to have a go. Enjoyed; engaged; getting something out because they were listening…"

Although this type of approach is not uncommon in a primary setting, it was acknowledged that curriculum enrichment activities such as these are less common in mathematics. This project has effectively exemplified how this can be done.

Conclusions and next steps

We have argued that maths-rich play-based learning that is clearly linked to specific learning objectives around reasoning, and that engages with two Phases of the Causal Connectivity Pedagogy Framework, can help pupils grasp the structures and connections with a mathematics topic. As teachers, we tend to underestimate pupils' mathematical thinking abilities. However pupil feedback from the project confirms that they are capable of reasoning, of using mathematics language, and of acquiring a deeper understanding of core knowledge that most staff expected.

By deliberately creating a situation in which children have to make sense of things for themselves, these lessons allow teachers to view learners through a different lens. In doing so, such projects have the potential to ensure all pupils have access to stretching activities, and the talents of young people – which may not be identified through more traditional routes – are identified. This validates the decision of the schools to deliver this project to all pupils within a particular year group rather than only their 'more able' cohort.

While there is a case for also offering the lesson series described here as an enrichment session as part of an after-school activity programme, they could also feature within mainstream curriculum time. This would ensure that all pupils have the opportunity to be stretched and challenged, and have the opportunity to benefit from learning through maths-rich play-based activities as part of their core curriculum offer.

Acknowledgement

We would like to thank all of the teachers who took part in the Snowflake Bentley Project during the 2017/18 academic year for their expertise and their valuable contributions.

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